MATHEMATICS APPLICATIONS

MAWA Semester 1 (Unit 3) Examination 2020

Calculator-Assumed

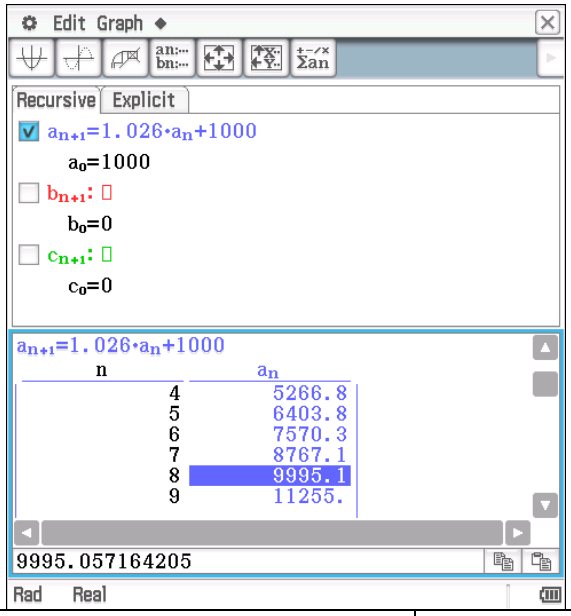
Marking Key

The release date for this exam and marking scheme is 12th June.

Section Two: Calculator-assumed (104 Marks)

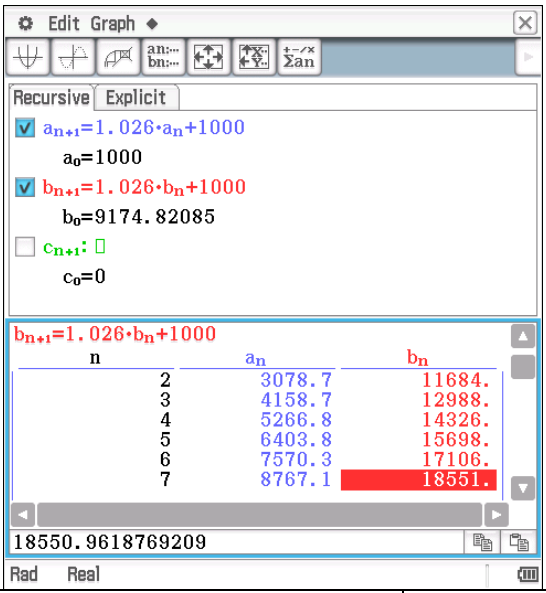
Question 7 (4 marks)

Question 7 (a) (2 marks)

Solution
The account grew to more than \$10000 in the 9th year. The balance in the year just before was \$9995.06. 

Marking key/mathematical behaviours	Marks
• indicates method used, such as stating the sequence applied	1
• states the correct year and balance requested.	1

Question 7 (b) (2 marks)

Solution
At the 18th anniversary, the account balance is \$24174.82. Withdrawal of \$15000, leaves \$9174.82 in the account. Seven years later the balance will be \$17550.96. (\$18550.96 - \$1000). 

Marking key/mathematical behaviours	Marks
• indicates method used, such as stating the new sequence applied	1
• states the correct balance on the 25th anniversary.	1

Question 8 (10 marks)

Question 8 (a) (3 marks)

Solution	
The sequence is a GP, so $T_n = ar^{n-1} \Rightarrow \frac{T_{10}}{T_4} = \frac{ar^9}{ar^3} = 64$ $\Rightarrow r^6 = 64 \Rightarrow r = \pm 2$ $\therefore$ The first four terms are either 1.6,3.2,6.4,12.8 or, 1.6, -3.2, 6.4, -12.8.	
Marking key/mathematical behaviours	Marks
• determines $r = 2$ • determines $r = \pm 2$ • states both sets of the first four terms (follow through if only states $r = 2$, that is, the correct terms of the positive sequence is awarded 2 marks total)	1 1 1

Question 8 (b) (3 marks)

Solution	
$n = 0,\ T_2 = T_1 - \frac{3(0)}{2} = 8 - 0 = 8$ $n = 1,\ T_3 = T_2 - \frac{3(1)}{2} = 8 - 1.5 = 6.5$ $n = 2,\ T_4 = T_3 - \frac{3(2)}{2} = 6.5 - 3 = 3.5$ $\therefore$ the first four terms are 8, 8, 6.5, 3.5	
Marking key/mathematical behaviours	Marks
• determines $T_2 = 8$ • determines terms 3 and 4 correctly • states the correct first 4 terms	1 1 1

Question 8 (c) **(4 marks)**

Solution

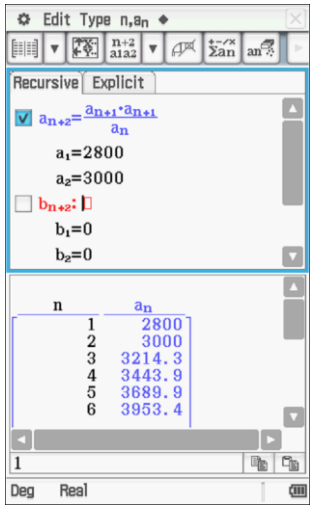
(i)

n	1	2	3	4	5	6
T_n	2800	3000	3214	3444	3690	3953

T_1 and T_2 are given.

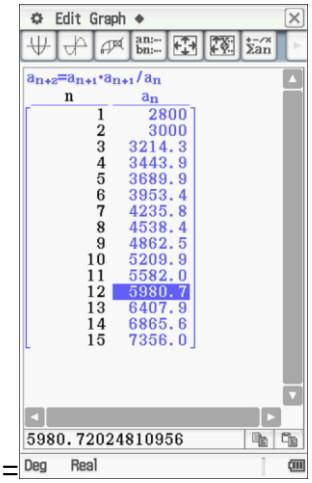
$$T_3 = T_2\left(\frac{T_2}{T_1}\right) = 3\times\left(\frac{3}{2.8}\right) = 3214 \text{ (rounded to nearest metre)}$$

etc. or using CAS



(ii) James walked 2800 m on day one.

More than twice = 5600 m. Hence from the CAS screen this will be on the 12th day of the program. T_{12}=5981 m



Marking key/mathematical behaviours	Marks
• completes T_1, T_2 and T_3 correctly	1
• completes the rest of the table correctly (and rounds correctly)	1+1
• states the 12th day.	1

Question 9 **(10 marks)**

Question 9 (a) **(4 marks)**

Solution	
(i) The explanatory variable is Area (ii) Whilst the Percentage Representation by Children across Areas is similar for the Population of WA (26%,27%) there is a marked increase in the Percentage Representation by Children using aquatic centres in Regional areas (52%) compared with Metropolitan areas (28%) (iii) Regional centres may have less facilities/opportunities for other activities resulting in Aquatic Centres being more readily used. Many Regional Centres are located inland whereas many Metropolitan Centres would be coastal. Those living in coastal areas may frequent the beach instead of Aquatic Centres resulting in higher % representation in Regional centres. Other plausible reason and explanation.	
Marking key/Mathematical behaviours	Marks
(i) • states Area	1
(ii) • states similarity in distribution of population across Areas of the state • notes marked difference between Metropolitan and Regional areas for Patronage by Children	1 1
(iii) • states a plausible reason with explanation	1

Question 9 (b) **(6 marks)**

Solution	
(i) $\frac{12}{36} \times 100\% \approx 33\%$ (ii) (iii) As the size of the facility increases the % costs per patron decrease. This is the case for both Regional and Metropolitan centres	

	Type 1	Type 2	Type 3
Regional	**62%**	**30%**	8%
Metropolitan	42%	33%	**25%**

Marking key/Mathematical behaviours	Marks
(i) • shows correct numerator • shows correct denominator and calculates percentage	1 1
(ii) • correctly calculates one Regional Percentage • correctly calculates three percentages	1 1
(iii) • identifies a relevant association • explains the association in context	1 1

Question 10 (14 marks)

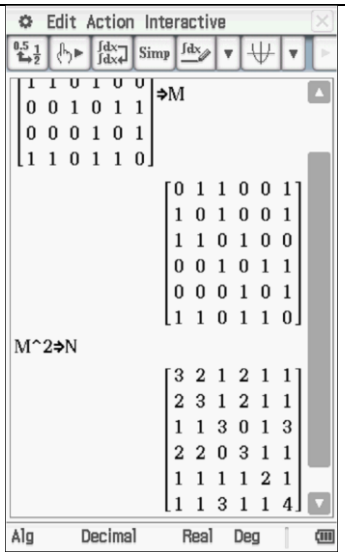
Question 10 (a) (i) (3 marks)

Solution						
	A	B	C	D	E	F
A	0	1	1	0	0	1
B	1	0	1	0	0	1
C	1	1	0	1	0	0
D	0	0	1	0	1	1
E	0	0	0	1	0	1
F	1	1	0	1	1	0

Marking key/mathematical behaviours	Marks
• uses the correct dimensions for the matrix	1
• completes the matrix with '0's and '1's and has most entries correct	1
• completes all entries correctly	1

Question 10 (a) (ii) (2 marks)

Solution						
	A	B	C	D	E	F
A	3	2	1	2	1	1
B	2	3	1	2	1	1
C	1	1	3	0	1	3
D	2	2	0	3	1	1
E	1	1	1	1	2	1
F	1	1	3	1	1	4



Marking key/mathematical behaviours	Marks
• clearly sets out the new 6x6 matrix	1
• correctly completes the matrix	1

Questions 10 (a) (iii) **(2 marks)**

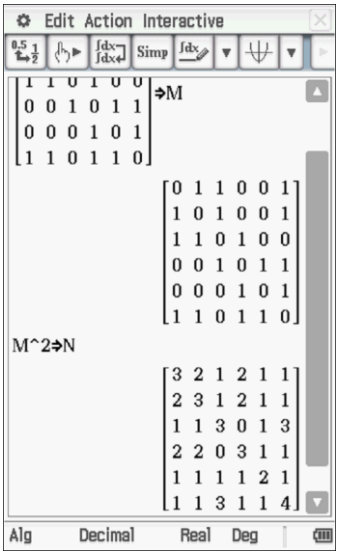
Solution	
Cells $M^2(3,4)$ and $M^2(4,3)$ both have a value equal to zero Cells $M^2(3,4)$ and $M^2(4,3)=0 \Rightarrow$ that it is not possible to fly to or from cities C and D via just one other city.	
Marking key/mathematical behaviours	**Marks**
• identifies the two cells in M^2	1
• explains clearly that $M^2(3,4)$ and $M^2(4,3)=0$ implies no two leg flights	1

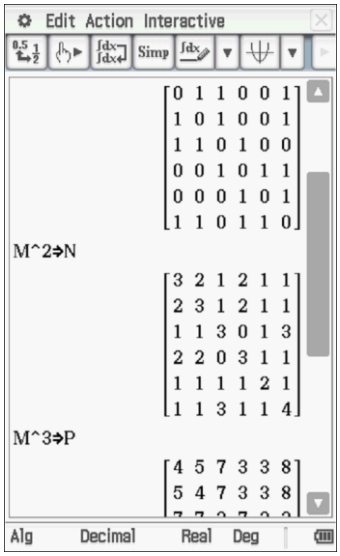
Questions 10 (a) (iv) **(2 marks)**

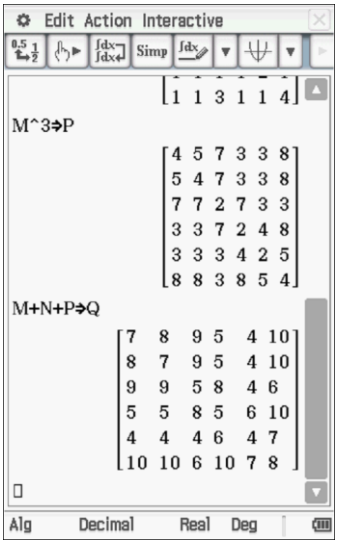
Solution	
Cells M(3,4) and M(4,3) both have a value equal to one M(3,4) and M(4,3) = 1 $\Rightarrow$ there are direct flights between cities C and D	
Marking key/mathematical behaviours	**Marks**
• states the values of the two cells in M	1
• explains clearly that M(3,4) and M(4,3) = 1 implies a direct flight	1

Question 10 (b)(i) **(3 marks)**

Solution







Marking key/mathematical behaviours	Marks
• lists M, M^2 and M^3 as the appropriate matrices	1
• calculates M^3 correctly	1
• calculates $M+M^2+ M^3 = P$	1

Question 10 (b)(ii) **(2 marks)**

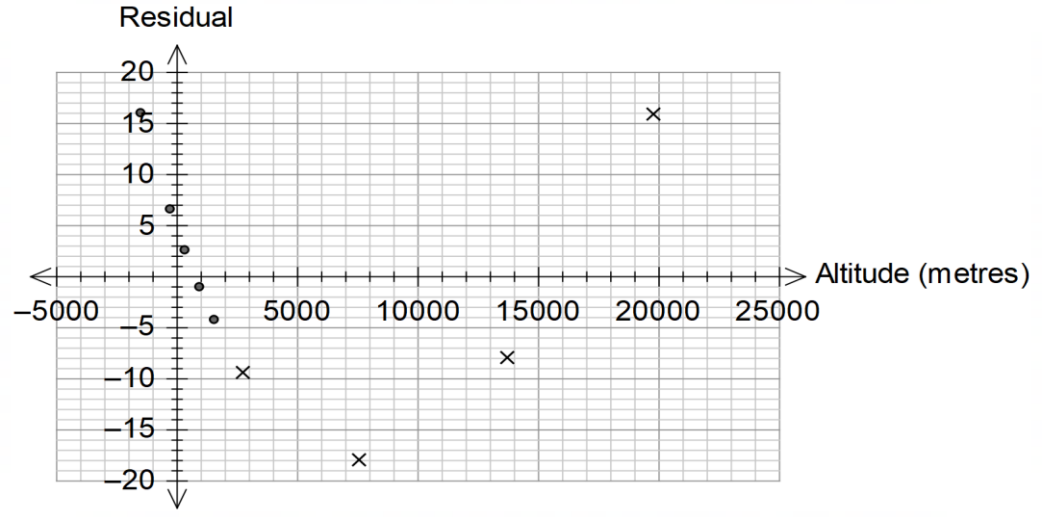
Solution	
(ii) Total = 7+7+5+5+4+8 =32	
Marking key/mathematical behaviours	Marks
• lists the elements of the leading diagonal of matrix P • calculates the sum of these elements	1 1

Question 11 **(14 marks)**

Question 11 (a) **(8 marks)**

Solution	
(i) $y = -0.0054x + 96.7087$ (ii) $x = -305, y = -0.0054 \times (-305) + 96.7087 \approx 98.36$ $x = 13716, y = -0.0054 \times 13716 + 96.7087 \approx 22.64$ (iii) y intercept is 96.7087 and this represents atmospheric pressure at sea level (iv) $r^2 = 0.9218$ hence 92.18% of the variation in Atmospheric Pressure can be explained by the variation in Altitude.	
Marking key/Mathematical behaviours	Marks
(i) • states correct linear equation • states y intercept correct to 4 decimal places • states gradient correct to 4 decimal places (ii) • states correct value for ***A*** • states correct value for ***B*** (iii) • states y intercept • relates value to sea level (iv) • states correct value	 1 1 1 1 1 1 1 1

Question 11 (b) **(6 marks)**

Solution
(i) ***C*** = 14.7 - 22.64 = -7.94 ***D*** = 5.6 – (-10.28) = 15.88 (ii)    (iii) A linear model is not appropriate as there is a pattern in the residuals.

Marking key/Mathematical behaviours	Marks
(i)	
• correctly determines ***C***	1
• correctly determines ***D***	1
(ii)	
See graph above	
• correctly graphs 2 points	1
• correctly graphs 4 points	1
(iii)	
• states a linear model is not appropriate	1
• states valid reason	1

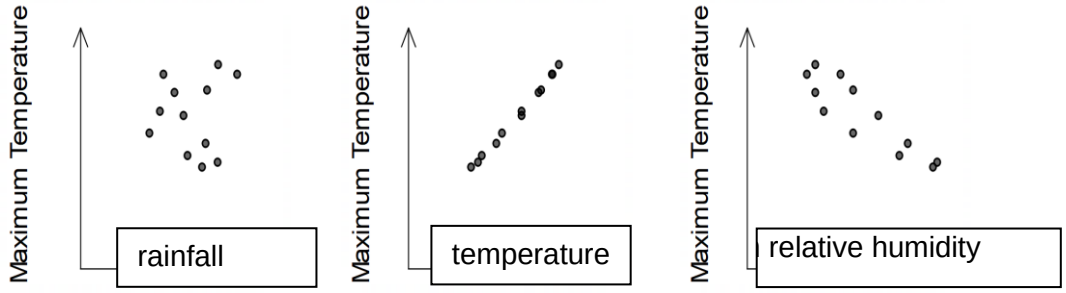
Question 12 **(12 marks)**

Question 12(a) **(1 mark)**

Solution
Generally as the humidity increases the Maximum temperature decreases. There is a negative relationship.

Marking key/Mathematical behaviours	Marks
• describes the relationship or states that it is negative	1

Question12 (b) **(2 marks)**

Solution


Marking key/Mathematical behaviours	Marks
• correctly labels exactly 1 graph	1
• correctly labels 3 graphs	1

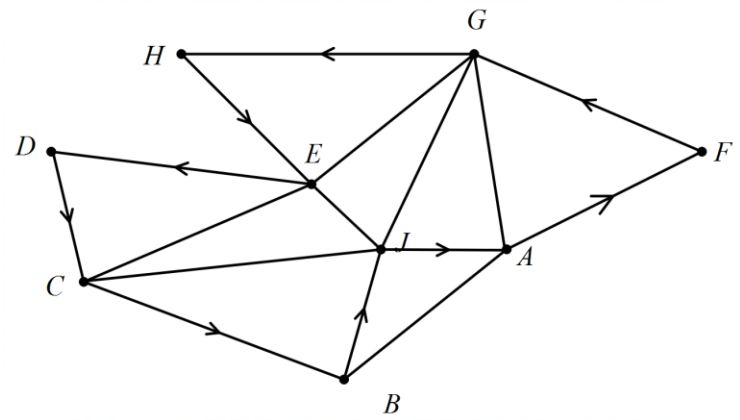
Question 12 (c) **(9 marks)**

Solution
(i) Yes. From the table and the scatter plot it is likely to be a good predictor of the maximum temperature as there is a strong correlation between the two. (ii) $x = 12, y = 1.0326 \times 12 + 0.2830 \approx 12.7^\circ$ This is an unreliable prediction as it involves extrapolation (iii) $r = \sqrt{0.9994} = 0.999699 \approx 0.9997$ There is a strong positive relationship between 3pm temperature and Maximum temperature. (iv) Since the equation is $y = 1.0326x + 0.2830$ every 1 degree increase in 3pm temperature results in a 1.0326° increase in the Maximum temperature. Hence the expected difference will be $15 \times 1.0326^\circ = 15.489^\circ \approx 15.5^\circ$

Marking key/Mathematical behaviours	Marks
(i)	
• states yes	1
• gives an explaination using the data	1
(ii)	
• substitutes $x = 12$ and states answer to 1 decimal place	1
• states unreliable	1
• states extrapolation	1
(iii)	
• states correlation coefficient to at least 4 decimal places	
• identifies strong and positive relationship	1
(iv)	1
• uses 1.0326°	
• states correct answer to 1 decimal place	1
	1

Question 13 (6 marks)

Question 13 (a) (2 marks)

Solution
 GHEDCBJAFG OR GFAJBCDEHG

Marking key/mathematical behaviours	Marks
• identifies a correct cycle	1
• lists the vertices in the correct order	1

Question 13 (b) (2 marks)

Solution
EG, AG, JG, CE, AB, JE and JC

Marking key/mathematical behaviours	Marks
• gives a list of appropriate edges mostly correct	1
• identifies all edges correctly	1

Question 13 (c) (2 marks)

CBJAFGHEDC OR CDEHGFAJBC

Marking key/mathematical behaviours	Marks
• identifies a correct cycle	1
• lists the vertices in the correct order	1

Question 14 (7 marks)

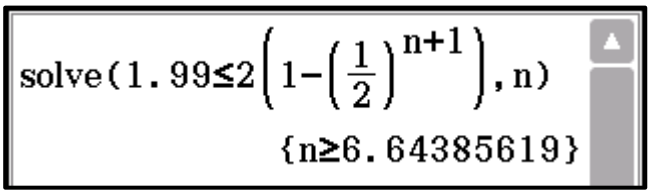
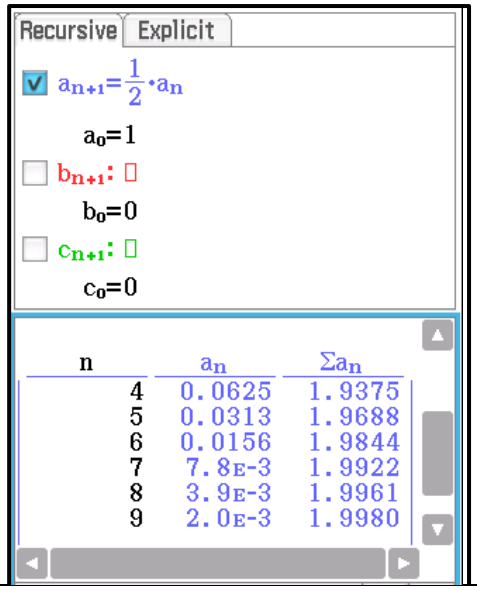
Question 14 (a) (1 mark)

Solution	
$T_1 = Area(\text{A0}) = 1$ $T_2 = Area(\text{A1}) = \frac{1}{2} \times T_1 = \frac{1}{2}$ $T_3 = Area(\text{A2}) = \frac{1}{2} \times T_2 = \frac{1}{4}$	
Marking key/mathematical behaviours	Marks
• determines the first 3 terms	1

Question 14 (b) (2 marks)

Solution	
$T_n = \frac{1}{2} \times T_{n-1},\ T_1 = 1$	
Marking key/mathematical behaviours	Marks
• states the correct recursive rule • specifies $T_1 = 1$	1 1

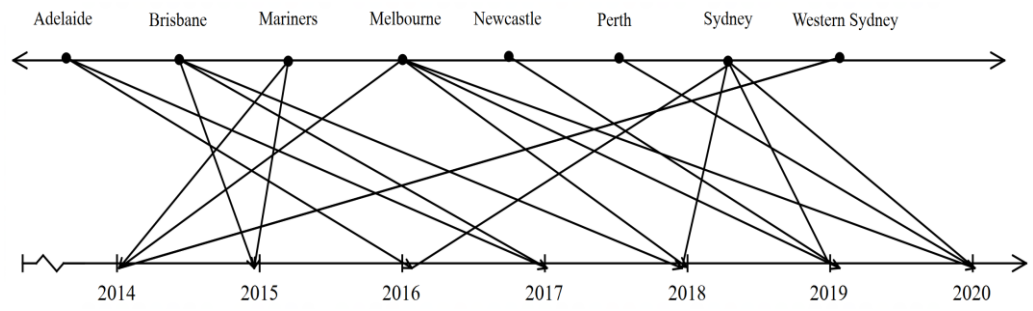
Question 14 (c) **(4 marks)**

Solution
Total Area = sum of the Areas of each sheet $\Rightarrow 1.99 = T_1 + T_2 + T_3 + ... + T_{n+1}$ The sequence is a GP with $a = 1$ and $r = \frac{1}{2}$ We want the smallest n such that the sum is just over 1.99 – so from CAS , by solving $1.99 \le \frac{1-\left(\frac{1}{2}\right)^{(n+1)}}{\frac{1}{2}}$ we get $n = 7$    Alternatively, set up the problem in the sequence in app defining the sequence and the sum as shown below to arrive at $n = 7$. 

Marking key/mathematical behaviours	Marks
• indicates that the sequence is a GP and states the parameters	1
• indicates that the sum of the sequence needs to >1.99	1
• states the correct $n = 7$	1
• indicates then paper size as A7 (ie. term 8 of the sequence)	1

Question 15 (8 marks)

Question 15 (a)

Solution
(i) 
(ii) Melbourne Victory

Marking key/mathematical behaviours	Marks
• sets up the two scales appropriately	2
• connects the bipartite elements clearly	1
• connects all the bipartite elements correctly	1
• states the correct club from the table	1

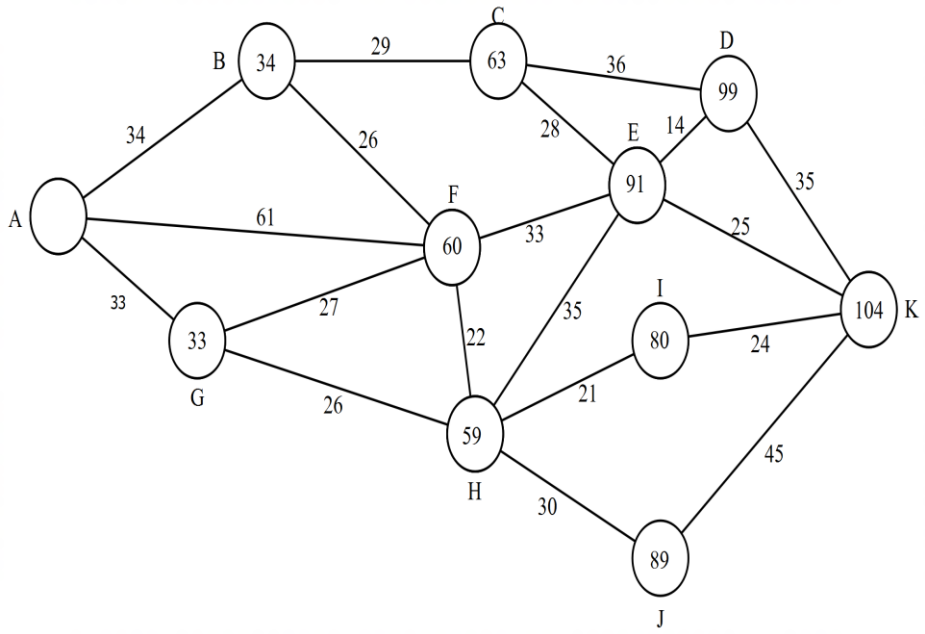
Question 15 (b) (3 marks)

Solution	
Let Matrix M have dimension $n \times n$ $\Leftrightarrow$ number of cells $= n^2$ Since the number of games $= 110$ $\Rightarrow n^2 = 110 + 11 \Rightarrow n = 11$ teams $\begin{bmatrix} (1,1) \ldots\ldots\ldots\ldots\ldots (1,11) \\ (2,1) \ldots\ldots\ldots\ldots\ldots (2,11) \\ \\ (10,1) \ldots\ldots\ldots\ldots (10,11) \\ (11,1) \ldots\ldots\ldots\ldots (11,11) \end{bmatrix}$	Alternate method: 11 teams each must play 10 other teams, at home which implies 110 matches altogether.

Marking key/mathematical behaviours	Marks
• uses appropriate logic	1
• clearly states the reasoning	1
• calculates the correct number	1

Question 16 (8 marks)

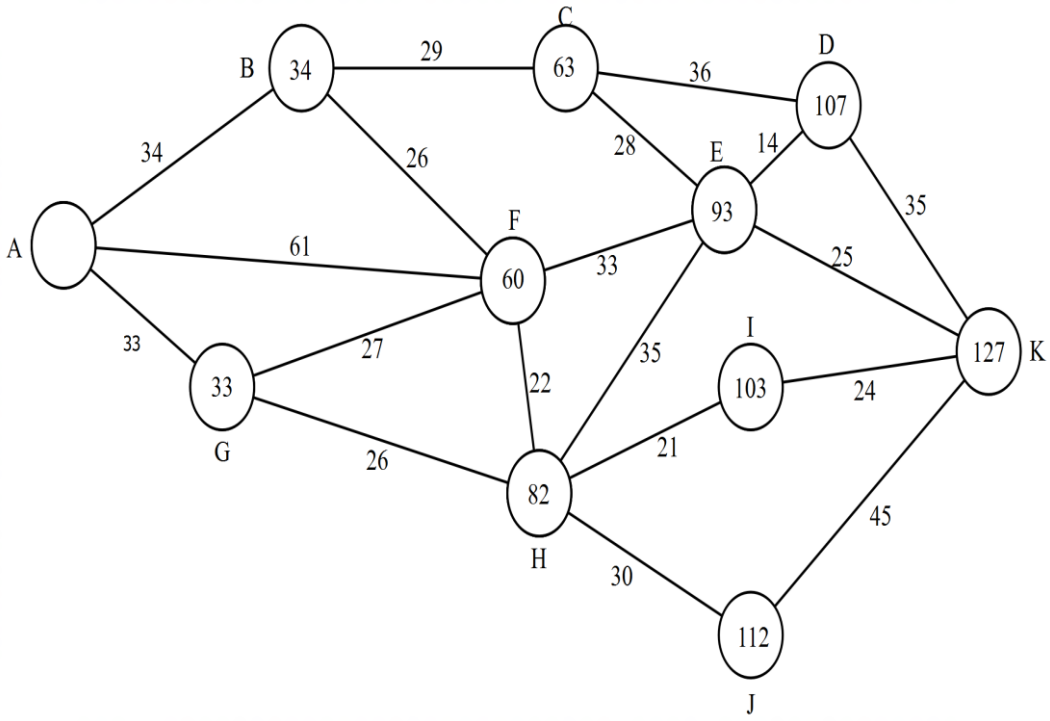
Question 16 (a) (4 marks)



AGHIK = 104 minutes

Marking key/mathematical behaviours	Marks
• Completes the algorithm by filling in values at the nodes	1
• Calculates at least 5 nodes correctly	1
• Calculates all 9 nodes correctly	1
• States the correct path	1

Question 16 (b)(i) **(2 marks)**

Solution
The modified path is shown below. 

Marking key/mathematical behaviours	Marks
• recalculates the times from F to G	1
• calculates these 6 nodes correctly	1

Question 16 (b)(ii) **(2 marks)**

Solution
Modified path = AFHIK Time = 127 minutes

Marking key/mathematical behaviours	Marks
• States the new path accurately	1
• States the new time accurately	1

Question 17 (11 marks)

Question 17 (a) (1 mark)

Depreciation amount will be 2.5% of \$160000 = \$4000 pa.

Marking key/mathematical behaviours	Marks
• calculates the correct depreciation	1

Question 17 (b) (4 marks)

Solution

Items	Original Value	Depreciation rate pa	Depreciation in 1st year	Depreciation in 2nd year
Carpets and floor coverings	\$9500	10%	\$950	\$855
Hot water system	\$2000	12%	\$240	\$211.20
Light fittings and window treatments	\$7500	15%	\$1125	\$956.25
Airconditioning system	\$6500	20%	\$1300	\$1040

Carpets: $\$9500 \times 10\% = \$950$

Light Fittings:

First year: $\$7500 \times 15\% = \$1125$

First year: $\$7500 - \$1125 = \$6375 \quad \therefore \text{Dep} = \$6375 \times 15\% = \$956.30$

Air conditioners: $(\$6500 - \$1300) \times 20\% = \$1040$

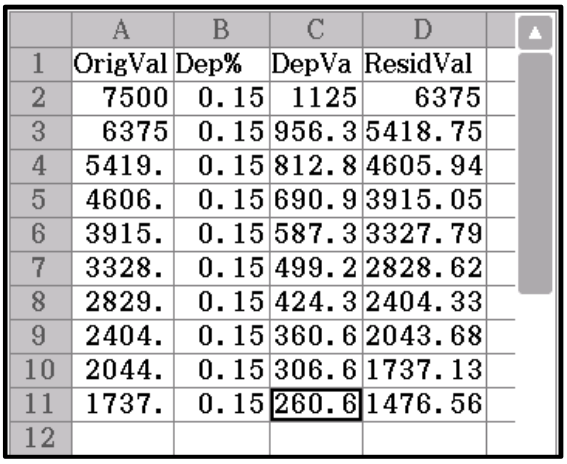
Alternatively, use a spreadsheet

eg. For Light fittings and window treatments below.

	A	B	C	D
1	OrigVal	Dep%	DepVa	ResidVal
2	7500	0.15	1125	6375
3	6375	0.15	956.3	5418.75
4	5419.	0.15	812.8	4605.94
5				

Marking key/mathematical behaviours	Marks
• indicating how to calculate depreciation for first year	1
• calculating both first year depreciations correctly	1
• indicating how to calculate depreciation for second year	1
• calculating both second year depreciations correctly	1

Question 17 (c) **(2 marks)**

Solution
The depreciation allowed in the tenth year is $260.60. Use a spreadsheet on CAS as shown below 

	A	B	C	D
1	OrigVal	Dep%	DepVa	ResidVal
2	7500	0.15	1125	6375
3	6375	0.15	956.3	5418.75
4	5419.	0.15	812.8	4605.94
5	4606.	0.15	690.9	3915.05
6	3915.	0.15	587.3	3327.79
7	3328.	0.15	499.2	2828.62
8	2829.	0.15	424.3	2404.33
9	2404.	0.15	360.6	2043.68
10	2044.	0.15	306.6	1737.13
11	1737.	0.15	260.6	1476.56
12				

Marking key/mathematical behaviours	Marks
• shows evidence of applying the diminishing value method correctly	1
• determines the correct answer	1